

# INF2034 Noções de Complexidade Computacional e sua relação com Lógica

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## Basics on Complexity Theory and Its Relationship to Logic

The goal of this course is to present and discuss the main definitions and results regarding computational complexity concepts. A second and equally important goal is to relate computational complexity with Logics. Recent techniques on compression of logical proofs and a recent proof theoretical demonstration of  $NP=PSPACE$  will be presented too. The course includes some discussion on Kolmogorov complexity and descriptive complexity, but at a more intuitive and informal approach.

Contents:

- Review on Computability:
  1. Turing Machines
  2. Recursive Functions, Recursion Theorems, Rices Theorem and Rogers Theorem.
  3. Kleenes Arithmetical Hierarchy.
  4. Ordinals and Gödels system  $T$  of transfinite types.
  5. Curry-Howard Isomorphism.
- Computational Complexity:
  1. Time and Space (resources) dynamic computational complexity estimates

2. The speedup theorem and asymptotic evaluation of algorithms
  3. Uniform diagonalization and the Gap theorem.
  4. Time constructive functions and its Hierarchy.
  5. Robustness of some computational models.
  6. Blums abstract setting to computational complexity.
  7. Natural classes of complexity in time and space, main conjectures.
- Logic and computational complexity:
    1. Polinomial Hierarchy and the conjecture  $NP =? P$ .
    2. Fagins theorems on logical characterization of P and NP.
    3. Proof complexity (length) and the CONP class.
    4. Kleenes hierarchy from the logical point of view.
    5. Oracles, relativization and Gill-Solovay technique on uniform diagonalization.
    6. NPI languages and other uniform diagonalization application.
  - Intuitionistic and Minimal logic and the PSPACE class.
  - The superpolynomial lower bound of propositional resolution.
  - Boolean circuits, Razborov Lower bound, Interpolation and short proofs.
  - Discussion on a recent approach to prove  $PSPACE=NP$ .
  - Kolmogorovs complexity and descriptive complexity.
  - Remarks on the relationship between complexity and proof theory as well as finite model theory.

References

## References

- [1] Christos Papadimitriou, Computational Complexity, Addison Wesley, 1994.
- [2] R. Epstein and W. Carnielli, Computability, 2009
- [3] Machtey and Young, General Theory of Algorithms, 1987.
- [4] J. Y. Girard , Proof Theory and Logical Complexity, 2003.
- [5] J. Krajicek., Bounded Arithmetic, Propositional Logic and Complexity Theory, 1995.
- [6] Bovet and Crescenzi, Introduction to the Theory of Complexity, 2002.
- [7] Neil Immerman, Descriptive Complexity, 1998.
- [8] L. Vytanyi, An Introduction to Kolmogorov Complexity and its Applications, 1997.
- [9] Lew Gordeev and Edward Hermann Haeusler, NP vs PSPACE , ArXiv e-prints, 1609.09562, 2016arXiv160909562G, <http://adsabs.harvard.edu/abs/2016arXiv160909562G>
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Evaluation of the students:

Exercises assignments and seminar presentations